



education

Department:
Education
PROVINCE OF KWAZULU-NATAL

MATHEMATICS P1

SEPTEMBER 2020

PREPARATORY EXAMINATION

MARKING GUIDELINE

**NATIONAL
SENIOR CERTIFICATE**

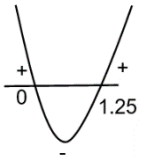
GRADE 12

MARKS: 150

TIME: 3 hours

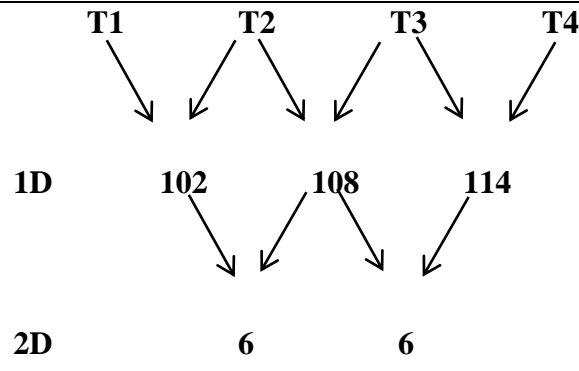
This marking guideline consists of 13 pages.

QUESTION 1

1.1.1	$x = 0$	A✓✓ 0	(2)
1.1.2	$-3x^2 + 8x + 7 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(8) \pm \sqrt{(8)^2 - 4(-3)(7)}}{2(-3)}$ $x = -0,69 \quad \text{or} \quad 3,36$	A✓ standard form CA✓ substitution in correct formula CA✓ CA✓ answers (penalize 1 mark if rounding off is incorrect-once here for entire paper)	(4)
1.1.3	$\sqrt{11 - x} - x = 1$ $\sqrt{11 - x} = x + 1$ $(\sqrt{11 - x})^2 = (x + 1)^2$ $11 - x = x^2 + 2x + 1$ $x^2 + 3x - 10 = 0$ $(x + 5)(x - 2) = 0$ $x = -5 \text{ or } x = 2$ $x = -5 \text{ n/a}$	A✓ Isolating surd A✓ squaring both sides CA✓ standard form CA✓ factors CA✓ answers and rejecting	(5)
1.1.4	$2 \cdot 3^x = 57 - 3^{x-2}$ $2 \cdot 3^x + 3^{x-2} = 57$ $3^x(2 + 3^{-2}) = 57$ $3^x \left(2 \frac{1}{9}\right) = 57$ $3^x = 27 = 3^3$ $x = 3$	A✓ Removing common factor CA✓ Simplifying bracket CA✓ $3^x = 27$ CA✓ answer	(4)
1.1.5	$4x^2 - 5x \leq 0$ $x(4x - 5) \leq 0$  $0 \leq x \leq \frac{5}{4}$	AA✓✓ factors CA✓ end points A✓ interval	(4)
			(4)

1.2	$2x + y = 3 \rightarrow (1)$ $y^2 = x^2 + y + x \rightarrow (2)$ From (1): $y = 3 - 2x \rightarrow (3)$ Substituting (3) into (2): $(3 - 2x)^2 = x^2 + (3 - 2x) + x$ $9 - 12x + 4x^2 = x^2 + 3 - 2x + x$ $3x^2 - 11x + 6 = 0$ $(3x - 2)(x - 3) = 0$ $x = \frac{2}{3} \quad \text{or} \quad x = 3$ $y = \frac{5}{3} \quad \text{or} \quad y = -3$	A✓ making y/x the subject CA✓ substitution CA✓ standard form CA✓ x - values CA✓ y values	(5)
			[24]

QUESTION 2

2.1	$T_k = 6k + 96 = 2022$ $k = 321$ Between the 321 st and 322 nd terms	A✓ equating k^{th} term to 2022 CA✓ k value CA✓ answer	(3)
2.2	 <p> $2a = 6 \quad a = 3$ $3a + b = 102 \quad b = 93$ $9a + 3b + c = 310 \quad c = 4$ $T_n = 3n^2 + 93n + 4$ </p> <p>OR</p> <p> $2a = 6 \quad a = 3$ $3a + b = 102 \quad b = 93$ $T_2 = 310 - 108 = 202$ $T_1 = 202 - 102 = 100$ $a + b + c = 100 \quad c = 4$ $T_n = 3n^2 + 93n + 4$ </p>	A✓ a value CA✓ b value CA✓ c value CA✓ answer	(4)
			(4)

	<p>OR</p> $2a = 6 \qquad a = 3$ $3a + b = 102 \qquad b = 93$ $T_2 = 310 - 108 = 202$ $T_1 = 202 - 102 = 100$ $T_n = T_1 + (n - 1)d_1 + \frac{(n - 1)(n - 2)}{2}d_2$ $T_n = 100 + (n - 1)(102) + \frac{(n - 1)(n - 2)}{2}(6)$ $T_n = 100 + 102n - 102 + 3n^2 - 9n + 6$ $T_n = 3n^2 + 93n + 4$ <p>OR</p> $2a = 6 \qquad a = 3$ $3a + b = 102 \qquad b = 93$ $T_2 = 310 - 108 = 202$ $T_1 = 202 - 102 = 100$ $T_n = \frac{n - 1}{2} [2a + (n - 2)d] + T_1$ $T_n = \frac{n - 1}{2} [2(102) + (n - 2)(6)] + 100$ $T_n = \frac{n - 1}{2} [204 + 6n - 12] + 100$ $T_n = \frac{n - 1}{2} [6n + 192] + 100$ $T_n = (n - 1)[3n + 96] + 100$ $T_n = 3n^2 + 93n + 4$	<p>OR</p> <p>A✓ a – value A✓ b – value</p> <p>CA✓ formula</p> <p>CA✓ answer</p> <p>OR</p> <p>A✓ a – value A✓ b – value</p> <p>CA✓ formula</p> <p>CA✓ answer</p>	<p>(4)</p> <p>(4)</p> <p>[7]</p>
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QUESTION 3

3.1.1	$T_n = \frac{1}{5}n + \frac{1}{5}$	A✓ common difference CA✓ answer	(2)
3.1.2	$S_n = \frac{n}{2} [2a + (n - 1)d]$ $S_{30} = \frac{30}{2} \left[2 \left(\frac{2}{5} \right) + (30 - 1) \left(\frac{1}{5} \right) \right]$ $= 99$	CA✓ $a = \frac{2}{5}$ and $d = \frac{1}{5}$ CA✓ substitution into formula CA✓ answer	(3)
3.2	$S_n = \frac{n}{2} [2a + (n - 1)d]$ $72710 = \frac{n}{2} [2(2) + (n - 1)(3)]$ $72710 = n/2 [3n + 1]$ $3n^2 + n - 145420 = 0$ $n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $n = \frac{-1 \pm \sqrt{(1)^2 - 4(3)(-145420)}}{2(3)}$ $n = 220 \quad \text{or} \quad -220.33$ <p style="text-align: center;"><i>n/a</i></p>	A✓ a - value and d - value CA ✓ simplifying bracket CA ✓ standard form CA ✓ substitution into formula CA ✓ answer and rejecting	(5)
			[10]

QUESTION 4

4.1	$a ; ar ; ar^2 ; ar^3 ; \dots$ $a + ar = 4(ar^2 + ar^3)$ $a(1 + r) = 4ar^2(1 + r)$ $a = 4ar^2 \quad r \neq -1$ $r^2 = \frac{1}{4} \quad ; \quad a > 0$ $r = \frac{1}{2} \quad ; \quad a > 0$	A✓ $a + ar$ A✓ $4(ar^2 + ar^3)$ A✓ factorising A✓ simplifying	(4)
4.2	$S_\infty = \frac{a}{1 - r} = 3$ $S_\infty = \frac{a}{1 - \frac{1}{2}} = 3$ $a = \frac{3}{2}$	A✓ equating sum to infinity to 3 CA✓ answer	(2)
			[6]

QUESTION 5

5.1	$x = -3$ and $y = -1$	$A\checkmark x = -3$ $A\checkmark y = -1$	(2)
5.2	y - intercept : $\left(0; -\frac{5}{6}\right)$ x - intercept: $\frac{1}{2(x+3)} - 1 = 0$ $2x + 6 = 1$ $x = -2\frac{1}{2}$ $\left(-2\frac{1}{2}; 0\right)$	$A\checkmark y$ - intercept $A\checkmark 2x + 6 = 1$ or $A\checkmark x + 3 = \frac{1}{2}$ $CA\checkmark x$ - intercept (co-ordinate form not needed)	(3)
5.3		$CA\checkmark x$ - intercepts $CA\checkmark y$ - intercept $CA\checkmark$ both asymptotes $A\checkmark$ shape	(4)
			[9]

QUESTION 6

6.1	A(0; 2)	A✓ answer (Must be in coordinate form)	(1)
6.2	$-x^2 + x + 2 = \frac{1}{2}x^2 - x$ $0 = \frac{3}{2}x^2 - 2x - 2$ $0 = 3x^2 - 4x - 4$ $(3x + 2)(x - 2) = 0$ $x = -\frac{2}{3} \quad \text{or} \quad x = 2$ $y = \frac{8}{9} \quad \text{or} \quad y = 0$ $C\left(-\frac{2}{3}; \frac{8}{9}\right) \quad \& \quad D(2; 0)$	A✓ equating CA✓ standard form CA✓ x – values CA✓ y – values CA✓ Writing in coordinate form	(5)
6.3	$x \leq -\frac{2}{3} \quad \text{or} \quad x \geq 2$	CA✓ $x \leq -\frac{2}{3}$ CA✓ $x \geq 2$	(2)
6.4	Length of PQ = $-x^2 + x + 2 - \left(\frac{1}{2}x^2 - x\right)$ $L = -\frac{3}{2}x^2 + 2x + 2$ $x = -\frac{b}{2a} = -\frac{2}{2\left(\frac{-3}{2}\right)} = \frac{2}{3}$ OR $L' = -3x + 2 = 0 \therefore x = \frac{2}{3}$ Maximum value of PQ $= -\frac{3}{2}\left(\frac{2}{3}\right)^2 + 2\left(\frac{2}{3}\right) + 2$ $= \frac{8}{3} = 2\frac{2}{3} = 2,67 \text{ units}$	A✓ subtraction of both graphs CA✓ equating in standard form CA✓ Axis of symmetry value or CA✓ Axis of symmetry value CA✓ AO value	(4)
6.5	$f(x) = -x^2 + x + 2$ $f'(x) = -2x + 1 = 3$ $x = -1$	AA✓✓ derivative and equating to 3 A✓ answer	(3)
6.6	Axis of symmetry: $x = \frac{1}{2}$ Maximum value: $y = -\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 2 = 2\frac{1}{4}$ $2 < k < 2\frac{1}{4}$	A✓ Axis of symmetry value CA✓ maximum value of f CA✓ end points A✓ interval	(4)
			[19]

QUESTION 7

7.1	Inverse: $x = \log_3 y$ $y = 3^x$	A✓ $x = \log_3 y$ A✓ $y = 3^x$ Answer only full marks	(2)
7.2	$y > 0$ or $y \in (0; \infty)$	A✓ answer	(1)
7.3	$2\log_3 x = -6$ $\log_3 x = -3$ $x = 3^{-3} = \frac{1}{27}$ $0 < x \leq \frac{1}{27}$	A✓ dividing by 2 A✓ writing in exponential form CA✓ end points A✓ interval Can be solved by log inequalities.	(4)
			[7]

QUESTION 8

8.1	$A = P(1 + i)^n$ $3P = P \left(1 + \frac{i}{12}\right)^{72}$ $i = 12(\sqrt[72]{3} - 1)$ $i = 0,1845$ Annual interest rate is 18,45 % p.a.	A✓ for using 3P and P A✓ $n = 72$ CA✓ making i the subject CA✓ answer	(4)
8.2.1	$P = \frac{x[1 - (1 + i)^{-n}]}{i}$ $192\ 000 = \frac{x \left[1 - \left(1 + \frac{0.12}{12}\right)^{-60}\right]}{\frac{0.12}{12}}$ $x = R4270,93$	A✓ value of n A✓ value of i CA✓ substitution into correct formula CA✓ answer	(4)
8.2.2	$P = \frac{x[1 - (1 + i)^{-n}]}{i}$ $4270,93 \left[1 - \left(1 + \frac{0.12}{12}\right)^{-15}\right]$ $= \frac{\frac{0.12}{12}}{\frac{0.12}{12}}$ $= R59216,72421$	A✓ Present value formula A✓ value of n CA✓ substitution into correct formula CA✓ answer	

	<p>OR</p> $A = P(1 + i)^n$ $A = 192\,000 \left(1 + \frac{0.12}{12}\right)^{45}$ $= R300\,443,6635$ $F = \frac{x[(1 + i)^n - 1]}{i}$ $F = \frac{4270,934 \left[\left(1 + \frac{0.12}{12}\right)^{45} - 1\right]}{\frac{0.12}{12}}$ $= R241\,226,9424$ <p>Balance on Loan</p> $= R300\,443,6635 - R241\,226,9424$ $= R59216,7211$	<p>OR</p> <p>A✓ Substitution into Compound Interest Formula</p> <p>CA✓ substitution into Future Value Formula</p> <p>CA✓ $A - F$</p> <p>CA✓ answer</p>	<p>(4)</p> <p>(4)</p>
		<p>[15]</p>	

QUESTION 9 (penalize 1 mark once for incorrect notation in this question)

<p>9.1</p>	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 - b(x+h) - (x^2 - bx)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - bx - bh - x^2 + bx}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{2xh + h^2 - bh}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{h(2x + h - b)}{h}$ $f'(x) = 2x - b$ <p>OR</p> $f(x+h) = (x+h)^2 - b(x+h)$ $f(x+h) = x^2 + 2xh + h^2 - bx - bh$ $f(x+h) - f(x) = 2xh + h^2 - bh$ $\frac{f(x+h) - f(x)}{h} = \frac{2xh + h^2 - bh}{h}$ $\frac{f(x+h) - f(x)}{h} = \frac{h(2x + h - b)}{h}$	<p>A✓ formula</p> <p>A✓ substitution</p> <p>CA✓ simplification of numerator</p> <p>CA✓ factorization</p> <p>CA✓ answer</p> <p>OR</p> <p>A✓ value of $f(x+h)$</p> <p>CA✓ simplification</p> <p>CA✓ factorization</p>	<p>(5)</p>
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	$f(-3) = (-3)^3 + 6(-3)^2 - 15(-3) + d$ $f(-3) = -27 + 54 + 45 + d$ $0 = 72 + d$ $-72 = d$ OR $f(x) = x^3 + 6x^2 - 15x + d$ $f(-3) = (-3)^3 + 6(-3)^2 - 15(-3) + d$ $f(-3) = -27 + 54 + 45 + d$ $0 = 72 + d$ $-72 = d$	A✓ substitution of $x = -3$ A✓ d – value A✓ anti derivative A✓ substitution A✓ simplification A✓ d – value	(4)
10.4	$x = -5 :$ $y = (-5)^3 + 6(-5)^2 - 15(-5) - 72 = 28$ $(-5 ; 28)$ maximum point $x = 1 :$ $y = (1)^3 + 12(1)^2 - 15(1) - 72 = -74$ $(1 ; -74)$ minimum point	CA✓ y – value CA✓ maximum point CA✓ y – value CA✓ minimum point	(4)
10.5	$3x^2 + 12x - 15 = t$ $3x^2 + 12x - 15 - t = 0$ $\Delta = b^2 - 4ac = 0$ $\Delta = (12)^2 - 4(3)(-15 - t) = 0$ $144 + 180 + 12t = 0$ $12t = -324$ $t = -27$	A✓ equating derivative to gradient of tangent A✓ standard form A✓ discriminant = 0 A✓ substitution A✓ t – value	(5)
			[20]

QUESTION 11

11.1	$Area = x^2 + 3x^2 + 4xh$ $Area = 4x^2 + 4xh$ $V = x^2h = 1000$ $h = \frac{1000}{x^2}$ $A = 4x^2 + 4x\left(\frac{1000}{x^2}\right)$ $A = 4x^2 + \frac{4000}{x}$	A ✓ Total Surface Area $A✓ h = \frac{1000}{x^2}$ A✓ Substitution for h	(3)
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<p>11.2</p>	$A = 4x^2 + 4000x^{-1}$ $A' = 8x - 4000x^{-2} = 0$ $8x = \frac{4000}{x^2}$ $x^3 = 500$ $x = \sqrt[3]{500} = 7,94 \text{ cm}$ $h = \frac{1000}{(7,94)^2} = 15,86 \text{ cm}$ <p>Alternatively if learners used: $A = 4x^2 + 400x^{-1}$</p> $A' = 8x - 400x^{-2} = 0$ $8x = \frac{400}{x^2}$ $x^3 = 50$ $x = \sqrt[3]{50} = 3,68 \text{ cm}$ $h = \frac{1000}{(3,68)^2} = 73,84 \text{ cm}$	<p>CA✓ derivative CA✓ derivative and equal to 0</p> <p>CA✓ $x^3 = 500$</p> <p>CA✓ x – value</p> <p>CA✓ substitution of x CA✓ h – value</p> <p>CA✓ derivative CA✓ derivative and equal to 0</p> <p>CA✓ $x^3 = 50$</p> <p>CA✓ x – value CA✓ substitution of x</p> <p>CA✓ h – value</p>	<p>(6)</p>
			<p>[9]</p>

QUESTION 12

				OUTCOMES	PROBABILITIES	
			0.98	P	AP	$49/250 = 0.196$
		A	0.02	NP	ANP	$1/250 = 0.004$
0.2			0.97	P	BP	$291/1000 = 0.291$
	0.3	B	0.03	NP	BNP	$9/1000 = 0.009$
	0.5		0.92	P	CP	$23/50 = 0.46$
		C	0.08	NP	CNP	$1/25 = 0.04$
A ✓			A ✓		A ✓	(4)

12.2.1	$\frac{9}{1000} = 0,009$	A✓ Answer	(1)
12.2.2	$P(NP) = P(ANP) + P(BNP) + P(CNP)$ $= 0,004 + 0,009 + 0,04$ $= 0,053 = \frac{53}{1000} = 5,3 \%$	A✓ formula CA✓ substitution CA✓ answer	(3)
			[8]

QUESTION 13

	Like ice-cream (L)	Do not like ice-cream(D)	Total
Boys (B)	65	30	95
Girls (G)	70	55	125
Total	135	85	220

13.1	68,42 %	AA✓✓answer	(2)
13.2	$P(BL) = \frac{65}{220} = \frac{13}{44} = 0,2955$	AA✓✓ $\frac{65}{220}$ or $\frac{13}{44}$ or 0,2955	(2)
13.3	$P(B) = \frac{95}{220} = \frac{19}{44} = 0,4318$ $P(B) \times P(L) = \frac{95}{220} \times \frac{135}{220}$ $= \frac{513}{1936} = 0,2650$ $P(B) \times P(L) \neq P(BL)$ Events are not independent.	CA✓ $\frac{95}{220}$ or $\frac{19}{44}$ or 0,4318 CA✓probability of product CA✓conclusion	(3)
			[7]

Total: 150